Sociometry
1977, Vol. 40, No. 1, 35-41

A Set of Measures of Centrality Based on Betweenness

LINTON C. FREEMAN
Lehigh University

A family of new measures of point and graph centrality based on early intuitions of Bavelas (1948) is introduced. These measures define centrality in terms of the degree to which a point falls on the shortest path between others and therefore has a potential for control of communication. They may be used to index centrality in any large or small network of symmetrical relations, whether connected or unconnected.

The recent article by Moxley and Moxley (1974) raised an important problem with respect to the measurement of centrality in social networks. The Moxleys were concerned with measuring centrality in the large, often unconnected, networks encountered in natural settings. The problem, as they defined it, was that the classical centrality measures of Bavelas (1950), Beauchamp (1965) and Sabidussi (1966) could not be used for unconnected networks. In each of these measures, the centrality of a point is a function of the sum of the minimum distances between that point and all others. Since all distance sums are infinite in unconnected networks, these measures are useful only in settings where connectivity can be assured.

The Moxleys' proposed solution for this problem was both arbitrary and *ad hoc*. They suggested that unconnected points be connected by an imaginary path with a length greater than that linking any pair of connected points in the network. The result is a crude ranking of the centrality of points and no index whatsoever of the overall centrality of the entire network. Moreover, since the rankings themselves are an artifact of a series of nonexistent connections, it is difficult to imagine what they might mean in terms of human communication.

The purpose of the present paper is to propose an alternative and more satisfactory solution to the problem posed by Moxley and Moxley. A new set of centrality measures will be introduced. They are based on the intuitions of Bavelas (1948) and others who originally used the centrality concepts in the context of the study of human communication. Here an attempt will be made to stick rather closely to these original ideas and to generate measures that are meaningful in communication terms.

POINT CENTRALITY: INTUITIVE BACKGROUND

The earliest intuitive conception of point centrality in communication was based upon the structural property of *betweenness*. According to this view, a point in a communication network is central to the extent that it falls on the shortest path between pairs of other points. This idea of point centrality was introduced by Bavelas (1948) in his first paper on the subject. He suggested that when a particular person in a group is strategically located on the shortest communication path connecting pairs of others, that person is in a central position. Other members of the network were assumed to be "responsive" to persons in such central positions who could influence the group by "withholding information (or) coloring or distorting it in transmission."

This same intuition was expressed by Shimbel (1953) who put it in quite different terms:

> "Suppose that in order for site i to contact site j, site k must be used as an intermediate station. Site k in such a network has a certain 'responsibility' to sites i and j.
>
> If we count all of the minimum paths which pass through site k, then we have a measure of the 'stress' which site k must undergo during the activity of the network. A vector giving this number for each number of the network would give us a good idea of stress conditions throughout the system."

Shaw (1954), however, returned to Bavelas' mode of expression. He said:

> "The relayer has it in his power to withhold information . . . or to refuse to pass on requests for information."

Such power, according to Shaw depends upon:

> ". . . the number of positions for which a given position serves as a relayer of information."

A somewhat different image of the same idea was employed by Cohn and Marriott (1958). They defined centers as the "nexes" that "bind and intertwine" the strands of networks and thus, it would seem, stand between other points. But instead of talking about blocking or distorting messages or stress, Cohn and Marriott emphasized the potential of such central points for binding the network together by coordinating the activities of other points.

Regardless of the imagery chosen, the importance of this conception of point centrality is in the potential of a point for *control* of information flow in the network. Positions are viewed as structurally central to the degree that they stand *between* others and can therefore facilitate, impede or bias the transmission of messages.

MEASURING POINT CENTRALITY

Although earlier intuitive statements conceived of point centrality in terms of betweenness, measures based on this concept have not been reported. Shaw (1954) included betweenness counts in a more complex measure, but provided no general procedures for measuring it. Such measurement, however, is rather straightforward.

Consider an unordered pair of points, $\{p_i, p_j\}$, $(i \neq j)$. Either p_i and p_j are unreachable from one another or there are one or more paths between them. In the latter case, each of the paths has a length equal to the number of edges contained in it.

Among the paths connecting p_i and p_j one or more have the shortest length: the geodesics. Moreover, if p_j is directly connected to p_i by an edge–if they are adjacent–there is only one geodesic (of length one) between them. If, however, a single geodesic connecting p_i and p_j has a length greater than one, other points fall on the path between p_i and p_j.

A point is considered to be central here to the degree that it falls between other points on their shortest or geodesic communication paths. A point falling between two others can facilitate, block, distort or falsify communication between the two; it can more or less completely control their communication. But if it falls on some but not all of the geodesics connecting a pair of points, its potential for control is more limited.

As an example consider the graph shown in Figure 1. There are two geodesics linking p_1 with p_3, one via p_2 and one via p_4. In such a case neither p_2 nor p_4 is between p_1 and p_3 in the strict graph theoretic sense and neither can totally control communications between these latter points. Both p_2 and p_4, however, may be viewed as having some potential for control.

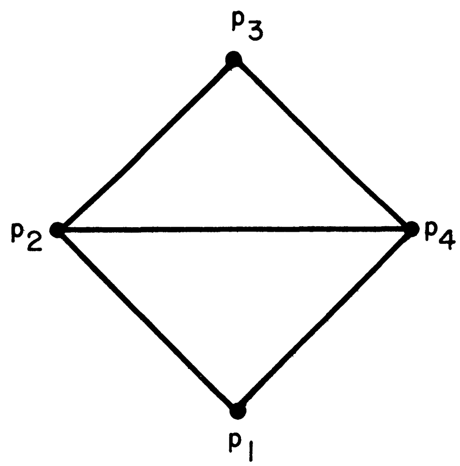

FIGURE 1
A Graph with Four Points and Five Edges

All this suggests that we need to generalize the graph theoretical notion of betweenness. Given a point, p_k, in a graph and an unordered pair of points, $\{p_i, p_j\}$ where $i \neq j \neq k$, we can define the partial betweenesss, $b_{ij}(p_k)$, of p_k with respect to (p_i, p_j) in the following way.

If p_i and p_j are not reachable from each other, p_k is not between them, so in that case let

$$b_{ij}(p_k) = 0.$$

If p_i and p_j are reachable, assume that they are indifferent with respect to the routing of their communication among alternative geodesics. Thus, the probability that a message passes along any particular geodesic among alternatives is equal to

$$\frac{1}{g_{ij}}$$

where g_{ij} = the number of geodesics linking p_i and p_j. The potential of point p_k for control of information passing between p_i and p_j then may be defined as the probability that p_k falls on a randomly selected geodesic connecting p_i and p_j. If $g_{ij}(p_k)$ = the number of geodesics linking p_i and p_j that contain p_k, then

$$b_{ij}(p_k) = \left(\frac{1}{g_{ij}}\right)\left(g_{ij}(p_k)\right) = \frac{g_{ij}(p_k)}{g_{ij}}$$

is the probability we seek. $b_{ij}(p_k)$ is the probability that point p_k falls on a randomly selected geodesic linking p_i with p_j.

In the illustration from Figure 1, p_2 and p_4 each have a probability of 1/2 of falling between p_1 and p_3. In general, if p_k falls on the only geodesic between p_i and p_j or if p_k falls on all the geodesics linking p_i and p_j, then $b_{ij}(p_k) = 1$. In these cases p_k can control communication because it is a necessary link between p_i and p_j.

To determine the overall centrality of a point, p_k, we need merely to sum its partial betweenness values for all unordered pairs of points where $i \neq j \neq k$:

$$C_B(p_k) = \sum_{i<j}^{n} \sum^{n} b_{ij}(p_k),$$

where n = the number of points in the graph.

The sum, $C_B(p_k)$, is an index of the overall partial betweenness of point p_k. Whenever p_k falls on the only geodesic connecting a pair of points, $C_B(p_k)$ is increased by 1. When there are alternative geodesics, $C_B(p_k)$ is increased in proportion to the frequency of occurrence of p_k among those alternatives.

Both locating and counting geodesics become tedious and difficult as the networks increase in size. Fortunately, however, matrix methods for both of these tasks are detailed in Harary *et al.* (1965:134-141). They are based in part upon an algorithm derived by Flament (1963) for finding the set of all geodesics connecting a pair of points. These methods permit the development of a simple computer program to calculate $C_B(p_k)$.

$C_B(p_k)$ indexes the potential of a point for control by counting its opportunities for control. It is the simplest and in many cases probably the most useful betweenness-based measure of centrality.

Since $C_B(p_k)$ is essentially a count, its magnitude depends upon two factors: (1) the arrangement of edges in the graph that define the location of p_k with respect to geodesics linking pairs of points; and (2) the number of points in the graph. Leavitt (1951) argued that for certain classes of substantive problems it is desirable to create a measure that eliminates the impact of the number of points from the measure.

Consider, for example, a point p_i in a graph containing five points. Let us say p_i has a value, $C_B(p_i) = 6$. On the other hand, assume a point, p_j, in a graph of 25 points where $C_B(p_j) = 6$. Both p_i and p_j have the same potential for control in absolute terms—they can facilitate or inhibit the same number of communications. However, they differ markedly in their relative potential for control within their respective networks. p_i can dominate more than half of the communications between pairs of points in its graph, while p_j can control only slightly more than one percent. To the degree that this potential for control is perceived as relative by participants in networks, p_i and p_j are in quite different positions with respect to centrality. What is needed in this context is a measure that is relative to its maximum value in terms of the number of points in the graph.

Consider S, a totally disconnected graph with n = the number of points ($n \geqslant 3$) and m = 0, the number of edges. For such a graph let r = 0, the number of unordered pairs, $\{p_i, p_j\}$, where p_i and p_j are mutually reachable, and $C_B(p_k) = 0$, the centrality index of a point, p_k.

Now if we add an edge to S, m = 1 and r = 1, but still $C_B(p_k) = 0$, since with only one edge, no point can fall on a path between any others.

However, when we add a second edge and let m = 2, it can be added either such that r = 3 and C_B (p_k) = 1 for a point if there is a connection with the previous edge as in Figure 2, or such that r = 2 and C_B (p_k) = 0 for all points as in Figure 3. Figure 2, then, shows a point, p_k, that falls on a path between p_i and p_j; this is the most central graph possible with m = 2.

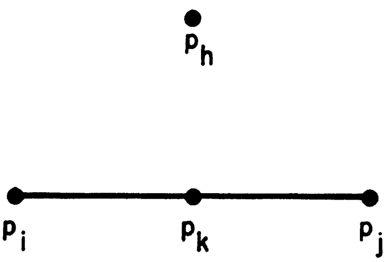

FIGURE 2
A graph with n = 4, m = 2, r = 3 and C_B (p_k) = 1

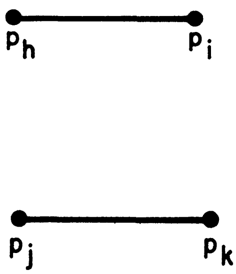

FIGURE 3
A graph with n = 4, m = 2, r = 2, and C_B (p_k) = 0

When successive new edges are added, maximum centrality is maintained only if all new edges are connected to the previous center point, p_k. This will be true until there are n −1 edges linking p_k with every other point in S. Under these conditions each point is reachable from all others either directly (in the case of p_k itself) or through p_k; S is connected.

Since all points are reachable there are n (n −1)/2 paths connecting the unordered pairs in S. Of these n − 1 are connected to p_k so the number of paths connecting pairs of points where p_k falls on the path between them is

$$C\,(p_k) = \frac{[n\,(n-1)]}{2} - [n-1] = \frac{n^2 - 3n + 2}{2} \quad .$$

Any new edge added to S after this stage must directly link two points that previously were connected only through p_k. Each new edge will, therefore, define a new geodesic that will reduce C (p_k) by one. Thus, maximum point centrality can be obtained only when the number of edges equals n −1 and there exists a point, p_k, that falls on all geodesics of length greater than one. Such a graph is called a star or wheel (see Figure 4), and it defines the upper limit of C (p_k) as

$$\max C_B\,(p_k) = \frac{n^2 - 3n + 2}{2} \quad .$$

The relative centrality of any point in a graph, then, may be expressed as a ratio,

$$C'_B\,(p_k) = \frac{2C_B\,(p_k)}{n^2 - 3n + 2} \quad .$$

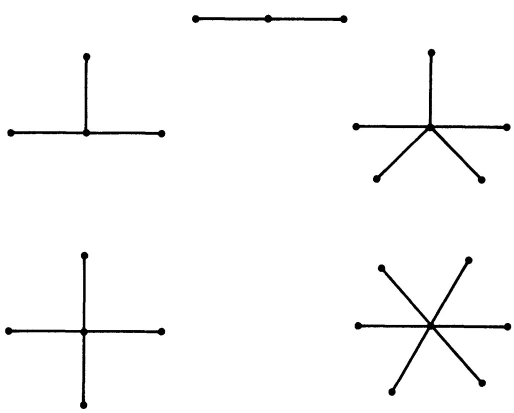

FIGURE 4
Graphs of Stars or Wheels for n = 3, 4, 5, 6, 7

Values of C'_B (p_k) may be compared between graphs. A wheel, for example, of any size will have a center point with C'_B (p_k) = 1; all other points will yield C'_B (p_k) = 0.

Both C_B (p_k) and C'_B (p_k) may be determined for any symmetric graph whether connected or not. Thus, these measures solve the problem raised by Moxley and Moxley (1974) of determining the centrality of points in unconnected graphs. Moreover, both measures take their maximum values only for points that are the centers of stars or the hubs of wheels like those shown in Figure 4. In the relative measure, $C'_B(p_k)$, all other points are scored as they compare with these maxima.

MEASURING GRAPH CENTRALITY

There are two distinct views on the meaning of the term *centrality* when it refers to a property of a whole network or graph. One of these, apparently based on graph theory, views a graph as exhibiting centrality to the degree that all of its points are central. Measures of this sort have been defined by Bavelas (1950) and Beauchamp (1965). They have limited utility and may be applied only to problems like the design of maximally efficient communication networks.

The alternative view leads to the development of measures of graph centrality based upon the dominance of one point. In this conception, a network is central to the degree that a single point can control its communication. Measures based upon this idea have been introduced by Leavitt (1951), Mackenzie (1966a) and Nieminen (1973, 1974). They turn out to be related empirically to a wide range of behavioral characteristics of communicating groups including perception of leadership, frequency of error, rate of activity, speed of organization and personal satisfaction or morale (Leavitt, 1951).[1]

What is needed here is a graph centrality measure of the second, more general, type. We can define $C'_B(p_k{}^*)$ as the largest centrality value associated with any point in the graph under investigation.

Then a natural measure of the dominance of the most central point is

$$C'_B = \frac{\sum_{i=1}^{n} [C_B(p_k{}^*) - C'_B(p_i)]}{n-1},$$

which is the average difference in centrality between the most central point and all others. C'_B varies between 0 and 1. Its value is 0 for all graphs of any size where the centralities of all points are equal. Its value is 1 only for the wheel or star. Thus C'_B is an expression of Mackenzie's prescription that:

> "A communication network is considered structurally centralized to the degree that the network approaches that of a wheel network and decentralized to the degree that the graph is an all-channel [complete]" (1966a).

USING THE BETWEENNESS-BASED MEASURES

The original application of the centrality idea was in the study of communication in small groups. Bavelas (1950), Leavitt (1951), Shaw (1954) and Goldberg (1955) all reported studies of speed, activity and efficiency in solving problems and personal satisfaction and leadership in small group settings. All of these variables were demonstrated to be related to centrality in some way.

More recently, the range of applications has been extended. Pitts (1965) examined the impact of centrality on urban growth. Czepiel (1974) used the concept in the study of the diffusion of a technological innovation in the steel industry. Rogers (1974) studied the emergence of two kinds of centrality in interorganizational relations. Cohn and Marriott (1958) used the idea in an attempt to explain political integration in Indian civilization. Both Beauchamp (1965) and Mackenzie (1966b) employed the concept of centrality in discussing the design of organizations. These several studies used perhaps a dozen different measures of centrality. While many were related, it is clear that there is little consensus on the solution to the problem of measuring centrality. Where, then, do three new measures fit into this picture?

The three measures, $C_B(p_k)$, $C'_B(p_k)$ and C'_B are more generally applicable than most of the alternatives. As has been illustrated, they are not limited to use in connected networks. The important question in considering applications—and the one that is most often neglected—involves considering the relevance of the particular structural attribute measured to the substantive problem being studied.

Thus, the use of these three new measures

[1] In his earlier paper, Nieminen (1973) added a weighting factor to his index of point dominance. This created a cumbersome measure, difficult to interpret, which resulted in a breakdown in the ability of his index to forecast empirical results. This problem was, however, eliminated in his subsequent paper (1974).

is appropriate only in networks where betweenness may be viewed as important in its potential for impact on the process being examined. Their use seems natural in the study of communication networks where the potential for control of communication by individual points may be substantively relevant.

Consider, for example, the relationship between point centrality and personal satisfaction in Leavitt's (1951) study of small group problem solving. Each participant had a piece of information necessary for the solution of a problem. Each could communicate only with designated others, and the problem could be solved only when all information was pooled.

Leavitt measured point centrality as a function of the lengths of paths or the distance between points. However, there is no reason to suspect that path length has any primary relationship with personal satisfaction in a task of this sort. There is no reason to believe that participants would be aware of, or even interested in, the distance between self and all other participants. On the other hand, participants cannot help but be sensitive to their roles as relayers or coordinators of information vital to the solution of the problem. To the degree that each stands between others, passes messages and thereby gains a sense of importance in contributing to a solution, he or she can be expected to be satisfied; the greater the betweenness, the greater his or her sense of participation and potency.

For the networks studied by Leavitt, however, path length and betweenness were highly related. To the degree that they are related, path length might also be expected to be related to satisfaction, but only as an artifact of the primary relationship. This seems to be true for the Leavitt data. Shaw (1954) has pointed out that the relationship between personal satisfaction and Leavitt's centrality index based on path length is not monotone. The relationship between personal satisfaction and $C_B(p_k)$, however, is.

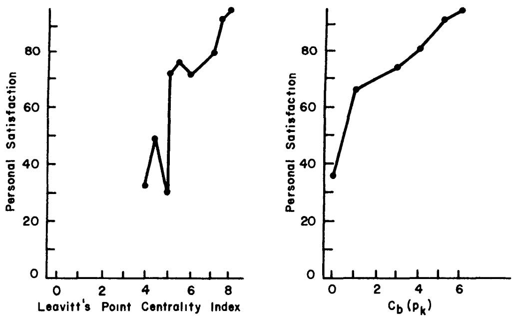

FIGURE 5
Relationship between personal satisfaction and two measures of point centrality (Leavitt's data)

These relationships, shown in Figure 5, suggest that $C_B(p_k)$ seems to be the better measure in this application and that the reasoning outlined above may be correct.

In any case, this illustrates the sort of reasoning that should be employed in selecting a centrality measure. The family of betweenness-based centrality measures introduced here should have a wide range of applications whenever there is reason to suspect that betweenness might be relevant to the substantive problem.

REFERENCES

Bavelas, Alex
1948 "A mathematical model for group structure." Applied Anthropology 7:16-30.
1950 "Communication patterns in task oriented groups." Journal of the Acoustical Society of America 57:271-282.

Beauchamp, Murray A.
1965 "An improved index of centrality." Behavioral Science 10:161-163.

Cohn, Bernard S. and McKim Marriott
1958 "Networks and centres of integration in Indian civilization." Journal of Social Research 1:1-9.

Czepiel, John A.
1974 "Word of mouth processes in the diffusion of a major technological innovation." Journal of Marketing Research 11:172-180.

Flament, Claude
1963 Applications of Graph Theory to Group Structure. Englewood Cliffs, N.J.: Prentice-Hall.

Goldberg, S. C.
1955 "Influence and leadership as a function of group structure." Journal of Abnormal and Social Psychology 51:119-122.

Harary, Frank, Robert Z. Norman and Dorwin Cartwright
1965 Structural Models: An Introduction to the Theory of Directed Graphs. New York: John Wiley and Sons, Inc.

Leavitt, Harold J.
1951 "Some effects of certain communication patterns on group performance." Journal of Abnormal and Social Psychology 46:38-50.

Mackenzie, Kenneth D.
1966a "Structural centrality in communications networks." Psychometrika 31:17-25.

1966b "The information theoretic entropy function as a total expected participation index for communication network experiments." Psychometrika 31:249-254.

Moxley, Robert L. and Nancy F. Moxley
1974 "Determining point-centrality in uncontrived social networks." Sociometry 37:122-130.

Nieminen, Juhari
1973 "On the centrality in a directed graph." Social Science Research 2:371-378.
1974 "On the centrality in a graph." Scandanavian Journal of Psychology 15:322-336.

Pitts, Forrest R.
1965 "A graph theoretic approach to historical geography." The Professional Geographer 17:15-20.

Rogers, David L.
1974 "Sociometric analysis of interorganizational relations: Application of theory and measurement." Rural Sociology 39:487-503.

Sabidussi, Gert
1966 "The centrality index of a graph." Psychometrika 31:581-603.

Shaw, Marvin E.
1954 "Group structure and the behavior of individuals in small groups." Journal of Psychology 38:139-149.

Shimbel, Alfonso
1953 "Structural parameters of communication networks." Bulletin of Mathematical Biophysics 15:501-507.

Sociometry
1977, Vol. 40, No. 1, 41-50

Equating Characteristics and Social Interaction: Two Experiments*

MURRAY WEBSTER, JR.
University of South Carolina

How do status characteristics which equate actors affect their performance expectations in face-to-face interaction? Differentiating characteristics are known to produce differential expectations through a burden of proof process—for example, females or blacks interacting with males or whites are assigned low ability expectations in a wide range of situations. Much of the literature assumes equating characteristics operate the same way, but this question has never been subjected to direct test. Two experiments are reported which indicate that equating characteristics do not lead to equal performance expectations in the situations studied.

How personal characteristics of actors affect their face-to-face interaction has long been one of the central concerns of social psychology. Hughes (1945) described the hypothetical interaction dilemma between a black female physician and a white male office-worker, and pointed out some of the relevant variables, and postulated some links between them. Personal characteristics were said to be significant because they were *diffuse evaluated* properties of actors; under certain unspecified conditions these characteristics were or were not seen by both parties as *relevant* to the interaction; and the interaction variables affected by characteristics seen as relevant all involve *subordination* and *superordination*.

Since Hughes' work, an enormous number of studies substantiate the observation that evaluated personal characteristics affect interaction in a great range of social situations. Torrance (1954) documented the effects of Air Force rank in laboratory groups; Whyte (1943) and Harvey (1953) described effects of informal status differences in boys' and adult gangs; and Strodtbeck (1958, 1965) analyzed effects of occupation and sex upon jury deliberation.

Theoretical and laboratory research by Berger and his colleagues (Berger *et al.*, 1966, 1972; Berger and Fisek, 1970) offers a perspective for analyzing such social situations as those above. All of these cases involve actors engaged in problem solving interaction. When actors are concerned with the solution

*Preparation of this report was made possible by a Summer Research Leave from the College of Humanities and Social Sciences, University of South Carolina.